Frequency Accuracy of the PicoPak Clock Measurement Module

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• PicoPak Frequency Accuracy

The frequency accuracy of the PicoPak clock measurement module can be limited by the precision of the software correction applied for its internal DDS frequency offset which is on the order of a pp10^9. That calculation is done with double-precision floating point arithmetic which has 16 digit precision. In most practical usage, the noise of the frequency sources and the module itself masks this small (pp10^{16}) error, as can medium-term temperature sensitivity. The value of the offset varies with the source frequency and corresponding DDS setting, as shown in Table I for several examples of the PicoPak DDS frequency offset at a nominal frequency of 10 MHz. The user interface values all agree with the exact numbers.

Table I. PicoPak DDS Frequency Offsets at 10 MHz Nominal

DDS Word Hex	Exact DDS Frequency Offset mHz and pp10^{10}	PicoPak User Interface (fDDS-fNom)/fNom	Remarks
15555557	+46.5661287307739257812	4.6566128731e-009	Maximum DDS setting
15555556	+18.6264514923095703125	1.8626451492e-009	
15555555	- 9.3132257461547851562	-9.3132257462e-010	Nominal DDS setting
15555554	-37.2529029846191406250	-3.7252902985e-009	
15555553	-65.1925802230834960937	-6.5192580223e-009	Minimum DDS setting

• PicoPak Frequency Offset with Coherent Signal and Reference Inputs

One would expect that a clock phase measurement instrument would show no average phase slope (frequency offset) with coherent signal and reference inputs derived from the same source. However, the DDS reference synthesizer in the PicoPak module introduces a very significant frequency offset that must be properly corrected for to achieve good accuracy. The PicoPak phase detector tracks the phase difference between the DDS reference and input signal. Because of the finite resolution of the DDS, there will be a frequency difference between these signals that causes their relative phase evolution to have a linear ramp. Fortunately, this phase slope is mathematically known and can be removed, but that removal is subject to inaccuracy caused by DDS phase quantization, DDS and source phase noise and finite numerical precision.

In the ideal noiseless, high numerical precision case, the DDS frequency offset is corrected for exactly. With fNom=10 MHz coherently applied to both the PicoPak signal and reference inputs, at the closest W=15555555 hex DDS tuning word and a x12 DDS multiplier, the DDS frequency, $fDDS=fRef\cdot 12\cdot W/2^{32}$, is offset by fDDS-fNom=-9.3132257461547851562… mHz, and that offset is corrected for by exactly the same amount. In actuality, the fDDS value calculated 16 digit precision is 9,999,999.9906867743000… compared with the exact value of 9,999,999.9906867742538…, a difference of 4.62e-18. The corresponding fractional frequency offset correction of $-9.3132257000\times10^{-10}$ is also correct to about 4.62e-18 and the numerical inprecision of the DDS frequency calculation is not a significant issue. In practice, source and internal noise, and thermal sensitivity mask the numerical inaccuracy even for long records. Coherent tests have shown residual phase slope (frequency offset) values of low pp10^{16} or less that are independent of the DDS tuning word.

- **PicoPak Scale Factor**

The PicoPak scale factor depends on the period of the DDS reference output to the phase detector and therefore varies with the DDS tuning word and the reference frequency input for a fixed tuning word. This, by itself, cannot cause a frequency offset, but could scale the magnitude of an offset. In a coherent case, the effect is very small; a 1pp10^9 reference frequency increase would change a 1pp10^{15} offset by only 1pp10^{24}. In a non-coherent measurement the primary effect of a reference frequency change is an equal and opposite change in the measured frequency.

- **PicoPak Phase Detector DC Offset**

The PicoPak uses a null seeking bang-bang servo to adjust the phase of the DDS to quadrature with respect to the signal, an average value of zero at the phase detector, which has an amplified sensitivity of about 0.31 mV/ps. The phase detector is biased at a reference voltage derived by a resistive voltage divider from the regulated 3.3 volt supply. Its output is amplified x11 by a DC amplifier and applied to one input of an analog comparator in the PIC microcontroller. The other comparator input is connected to a nearly identical stable reference voltage inside the PIC. Any DC offset in these components will cause a phase offset. That is of little consequence itself, but a rate of change of the phase offset will appear as a frequency change. While this is a contributor to PicoPak temperature coefficient, it is not possible for a thermally-induced phase ramp to continue indefinitely and thereby cause a steady-state frequency offset.

- **PicoPak Nominal Frequency**

The PicoPak uses a user-entered nominal frequency value, fNom, for several of its internal calculations. The main purpose of this quantity is to calculate fractional frequency values based on a nominal frequency set by the user who knows it best.

- **PicoPak Sampling Interval**

The apparent frequency offset of a phase record also depends on the measurement sampling interval, which, for the PicoPak module, is determined by dividing its 10 MHz reference using a dedicated hardware divider which cannot introduce an error.

- **PicoPak Phase Data Dynamic Range**

The PicoPak incremental phase data scatters around that corresponding to the fixed DDS offset value, and is therefore not subject to a dynamic range problem.

- **PicoPak Frequency Accuracy Tests**

The frequency accuracy of PicoPak module S/N 103 was measured during a 4-day, tau=1 second coherent run at 10 MHz at a nominal DDS setting of 15555555 hex. The resulting phase record, shown in Figure 1, had a slope corresponding to an average frequency offset of $+9.67 \times 10^{-17}$ as determined by its average phase slope.

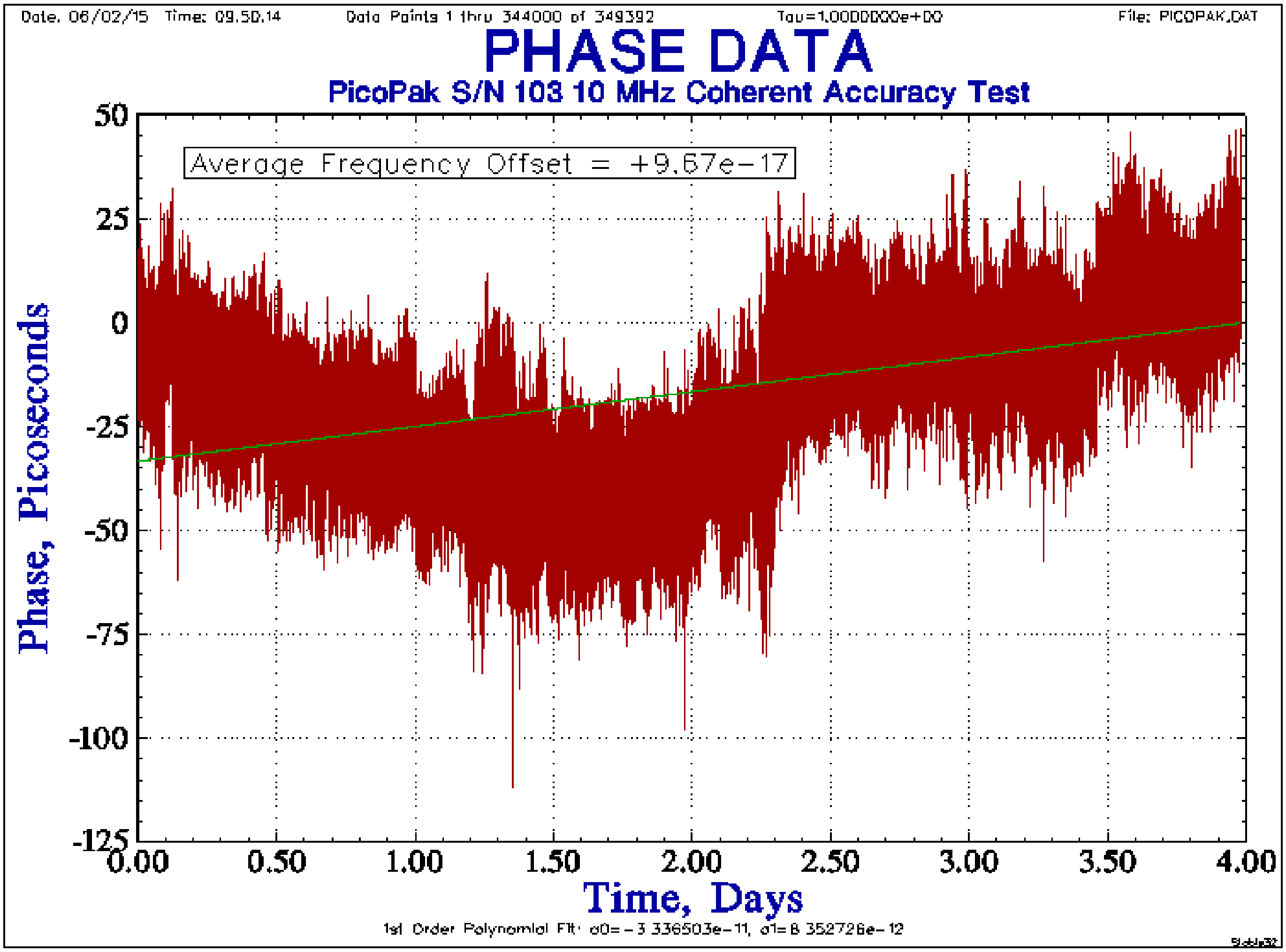

Figure 1. Phase Record of 4-Day Tau=1 Second Coherent 10 MHz PicoPak S/N 103 Run

The phase record shows normal PicoPak noise and slow variations most probably due to room temperature variations. Compared with those factors, the frequency inaccuracy is small.

Similarly, the frequency accuracy of PicoPak module S/N 105 was measured during a 30-day, tau=1 second coherent run at 10 MHz at a nominal DDS setting of 15555555 hex. The resulting phase record, shown in Figure 2, had a slope corresponding to an average frequency offset of -7.20×10^{-18} as determined by its overall phase slope. There were no outliers during this 1 month test.

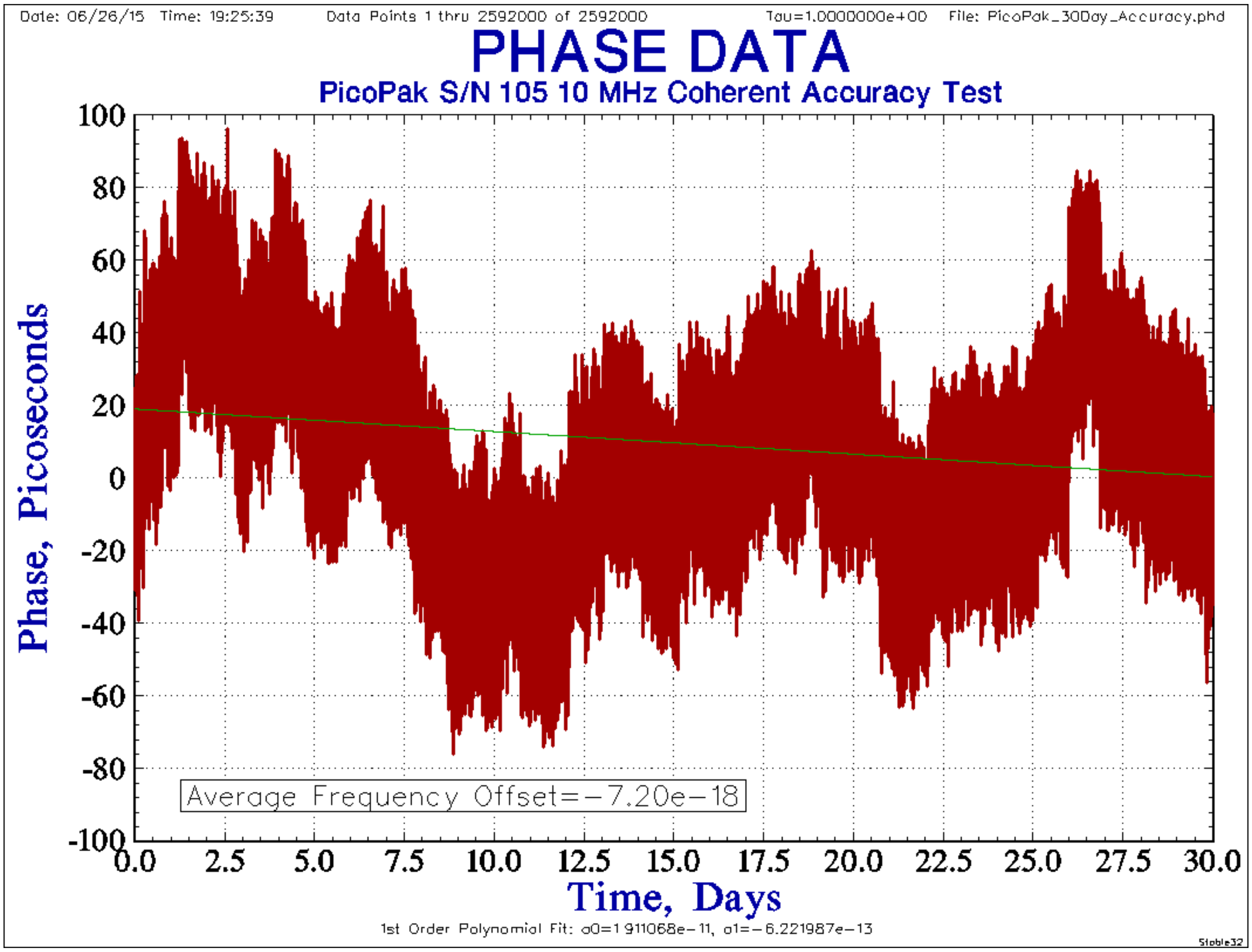

Figure 2. Phase Record of 30-Day Tau=1 Second Coherent 10 MHz PicoPak S/N 105 Run

The phase record shows normal PicoPak noise and cyclic variations most probably due to room temperature variations. Compared with those factors, the frequency inaccuracy is very small.

- **Conclusion**

We conclude that the PicoPak frequency accuracy is below a few pp10^{16}, is consistent with its resolution, noise, temperature sensitivity and other attributes, and is entirely adequate for its intended applications.

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